Suppose we have a dynamic system composed of m particles and possessing n degrees of freedom. Also, in addition to the r distinct generalized coördinates there are s superfluous ones. All these are bound together by k relations of the following form:

$$\sum_{\alpha=1}^{r} a_{\alpha\beta}\, dq_\alpha + \sum_{\alpha=r+1}^{r+s} p_{\alpha\beta} + b_\beta\, dt = 0 \quad \beta = 1 \ldots k \tag{1}$$

where $a_{\alpha\beta}$ and b_β are functions of the coördinates q_α and the time t and where $p_{\alpha\beta}$ represent linear, differential functions of the same. Hence, $n = r + s - k$. This equation indicates the type of the system: holonomic if s can be made equal to k, non-holonomic if s must be less than k, (unsolvable for $s > k$)

First we set up Gibbs' equation in relativistic form for the Cartesian coördinates of the particles:

$$\sum_{i=1}^{m} \sum_{j=1}^{3} \left[\mathrm{X}_{ij} - \frac{d}{dt} (\mathrm{M}_{ij}\, \dot{x}_{ij}) \right] \delta\, \ddot{x}_{ij} = 0 \tag{2}$$

where the symbols have their usual meanings. In order to transform this expression into one involving the generalized coördinates only, we make use of certain relations (that must be given) of the following type:

$$x_{ij} = x_{ij}\,(q_1 \ldots . \, q_{r+s},\, t)$$

Hence,

$$\delta\, \ddot{x}_{ij} = \sum_{\alpha=1}^{r+s} \frac{\partial\, \ddot{x}_{ij}}{\partial\, q_\alpha} \delta\, q_\alpha + \sum_{\alpha=1}^{r+s} \frac{\partial\, \ddot{x}_{ij}}{\partial\, \dot{q}_\alpha} \delta\, \dot{q}_\alpha + \sum_{\alpha=1}^{r+s} \frac{\partial\, \ddot{x}_{ij}}{\partial\, \ddot{q}_\alpha} \delta\, \ddot{q}_\alpha + \frac{\partial\, \ddot{x}_{ij}}{\partial\, t} \delta t$$

But the validity of equation (2) is conditioned by the vanishing of both δq_α and $\delta \dot{q}_\alpha$ at a given instant. Thus we obtain

$$\delta\, \ddot{x}_{ij} = \sum_{\alpha=1}^{r+s} \frac{\partial\, \ddot{x}_{ij}}{\partial\, \ddot{q}_\alpha} \delta\, \ddot{q}_\alpha \tag{3}$$

not all the $\delta \ddot{q}_\alpha$ are independent. Therefore we must consider the modification in (3) due to equations (1). The independence of the latter and their linearity enable us to solve for k of the coördinate-differentials, which can then be expressed linearly in terms of the remaining ones. Denoting the coördinates by Q_α after such a procedure, we have

$$dQ_\beta = \sum_{\gamma=1}^{n} A_{\beta\gamma}\, dQ_{k+\gamma} + B_\beta\, dt \quad \beta = 1 \ldots . k$$

Subjecting the variations to the same conditions as above, we obtain

$$\delta \ddot{Q}_\beta = \sum_{\gamma=1}^{n} A_{\beta\gamma}\, \delta \ddot{Q}_{k+\gamma} \quad \beta = 1 \ldots k$$

We now substitute these $\delta \ddot{Q}_\beta$ in (3). Hence,

$$\delta\, \ddot{x}_{ij} = \sum_{\gamma=1}^{n} C_{ij\gamma}\, \delta \ddot{Q}_{k+\gamma} \tag{4}$$

and equation (2) becomes

$$\sum_{i=1}^{m} \sum_{j=1}^{3} \left\{ \left[X_{ij} - \frac{d}{dt}(M_{ij}\, \dot{x}_{ij}) \right] \sum_{\gamma=1}^{n} C_{ij\gamma}\, \delta \ddot{Q}_{k+\gamma} \right\} = 0 \tag{5}$$

or

$$\sum_{\gamma=1}^{n} \left\{ \sum_{i=1}^{m} \sum_{j=1}^{3} \left(\left[X_{ij} - \frac{d}{dt}(M_{ij}\, \dot{x}_{ij}) \right] C_{ij\gamma} \right) \right\} \delta \ddot{Q}_{k+\gamma} = 0$$

But the $\delta \ddot{Q}_{k+\gamma}$ are arbitrary. Therefore

$$\sum_{i=1}^{m} \sum_{j=1}^{3} \left(\left[X_{ij} - \frac{d}{dt}(M_{ij}\, \dot{x}_{ij}) \right] C_{ij\gamma} \right) = 0 \quad \gamma = 1 \ldots n \tag{6}$$

These are the equations of motion.

As a matter of secondary interest, we shall now express these relations in a more compact—but less convenient—form. For

$$\sum_{i=1}^{m} \sum_{j=1}^{3} \left[\frac{d}{dt}(M_{ij}\, \dot{x}_{ij})\, C_{ij\gamma} \right] = \sum_{i=1}^{m} \sum_{j=1}^{3} \frac{M_{io}}{\left[1 - \left(\frac{\dot{x}_{ij}}{c} \right)^2 \right]^{3/2}} C_{ij\gamma}\, \ddot{x}_{ij}$$

where M_{io} is the rest-mass and c is the velocity of light.

But $\dot{x}_{ij}$ can be written

$$\dot{x}_{ij} = \sum_{\gamma=1}^{n} C_{ij\gamma}\, \dot{Q}_{k+\gamma} + C_{ij} \tag{7}$$

where C_{ij} represents the terms due to the explicit dependence on the time.

Likewise

$$\ddot{x}_{ij} = \sum_{\gamma=1}^{n} \dot{C}_{ij\gamma} \dot{Q}_{k+\gamma} + \sum_{\gamma=1}^{n} C_{ij\gamma} \ddot{Q}_{k+\gamma} + \dot{C}_{ij} \tag{8}$$

From (7) we conclude that $\dfrac{M_{io}}{\left[1-\left(\dfrac{\dot{x}_{ij}}{c}\right)^2\right]^{3/2}}$ is not a function of $\ddot{Q}_{k+\gamma}$.

And from (8) we note that $C_{ij\gamma}$ can be regarded as $\dfrac{\partial \ddot{x}_{ij}}{\partial \ddot{Q}_{k+\gamma}}$ (the derivative being purely formal). Let

$$\Gamma_\gamma \equiv \sum_{i=1}^{m} \sum_{j=1}^{3} X_{ij} C_{ij\gamma}$$

and

$$\Lambda \equiv \sum_{i=1}^{m} \sum_{j=1}^{3} \frac{1}{2} \frac{M_{io} \ddot{x}_{ij}^2}{\left[1-\left(\dfrac{\dot{x}_{ij}}{c}\right)^2\right]^{3/2}}$$

Then

$$\Gamma_\gamma = \frac{\partial \Lambda}{\partial \ddot{Q}_{k+\gamma}} \qquad \gamma = 1 \ldots . n \tag{9}$$

These equations can be identified with those of Appell for the non-relativistic case by a consideration of coördinate-velocities small with respect to the velocity of light. For, our Λ then reduces to his "energy of the accelerations."

BOTANY.—*The genus* Trichanthera.[1] EMERY C. LEONARD, U. S. National Museum. (Communicated by E. P. KILLIP.)

Until recently the genus *Trichanthera* of the Acanthaceae family was known from a single species, *T. gigantea*. It was fully described and finely illustrated by Humboldt and Bonpland as early as 1809[2] but was placed by them in the genus *Ruellia*. Their uncertainty as to the correctness of this course of action is shown by their remarks—"La plante que je viens d'écrire se trouve bien placée parmi les Acanthes;

[1] Published by permission of the Secretary of the Smithsonian Institution. Received September 3, 1930.

[2] Pl. Aequin **2**: 68. 1807.

mais il n'est pas facil de la rapporter à un des genères connus de cet ordre, ou d'en établir un nouveau qui offre des charactères bien tranchés. Dans cette incertitude, j'ai préferé la rapporter au genère Ruellia, plutôt que d'en établir un nouveau. Je donne à cette nouvelle plant le nom de Ruellia gigantea, parce que c'est un arbre très élevé."

The name *Trichanthera* was introduced ten years later when Kunth redescribed[3] this well-marked species and added the following observation, "Certe distincti generis ob stamina exserta, antheras pilosas et capsulae loculos dispermos. Fortasse Trichanthera nominandum."

In the present paper two species and one variety are described. These consist of attractive large-leaved shrubs or trees with silky reddish flowers crowded in terminal racemes or corymbs.

KEY TO THE SPECIES

Calyx lobes rounded; inflorescence racemose, secund; lower surface of leaf blades inconspicuously pubescent........................ **1. T. gigantea.**
Calyx lobes acute; inflorescence corymbose; lower surface of leaf blades conspicuously pubescent...............................**2. T. corymbosa.**

1. TRICHANTHERA GIGANTEA (Humb. & Bonpl.) Nees in DC. Prodr. **11**: 218. 1847.

Ruellia gigantea Humb. & Bonpl. Pl. Aequin. **2**: 68. pl. 102. 1807.
Trixanthera angularis Raf. Fl. Tellur. 146. 1838.

Shrub or tree up to 5 meters high (sometimes bushy and bearing adventitious roots); top round; twigs quadrate, the angles rounded, the tips minutely brown-tomentose; lenticels prominent, round, about 1 mm. in diameter; petioles 1 to 5 cm. long, channeled, glabrous or minutely pubescent; leaf blades ovate to oblong, the largest seen 26 cm. long and 14 cm. broad, acuminate at apex, narrowed at base, entire or undulate, glabrous except the veins and midrib, these prominent and more or less pubescent; inflorescence a terminal campact, secund panicle 5 to 15 cm. long and 4 to 5 cm. broad, brown-tomentose; bracts triangular, 3 mm. long; calyx 10 to 12 mm. long, brown-tomentose, the lobes erect, oblong, 7 to 10 mm. long, 5 mm. broad, rounded at apex; corolla 3 to 4 cm. long, red and glabrous proximally, yellowish and silky tomentose distally, red and glabrous within, the tube 1 to 1.5 cm. long, 6 mm. broad, sometimes slightly swollen or curved, the throat campanulate, the limb 2 to 3 cm. broad, the lobes oblong to oblong-ovate, 10 mm. long, 3 to 5 mm. broad; stamens exserted, the filaments 3 to 3.5 mm. long, pilose below, glabrous above, the anthers 6 mm. long, 3 mm. broad, bluntly apiculate at apex, bearded along the sutures, the hairs white and about 2 mm. long; ovary tomentose, 8-ovuled; style 4 to 5 cm. long, glabrous; stigma 2-lobed, one lobe vestigial, the other subulate, 2 mm. long; capsule oblong, 1.5 to 2 cm. long, 0.5 cm. broad, obtuse at apex, silky pubescent with closely appressed hairs, retinacula 3 mm. long, curved, truncate and erose at tip; mature seeds 1 to 4 in each capsule, lenticular, glabrous, 3 to 4 mm. in diameter.

Type locality: "In sylvis fluvii magdalenae prope Badillas," Colombia.

Specimens examined:

[3] H. B. K. Nov. Gen. & Sp. **2**: 243. 1817.

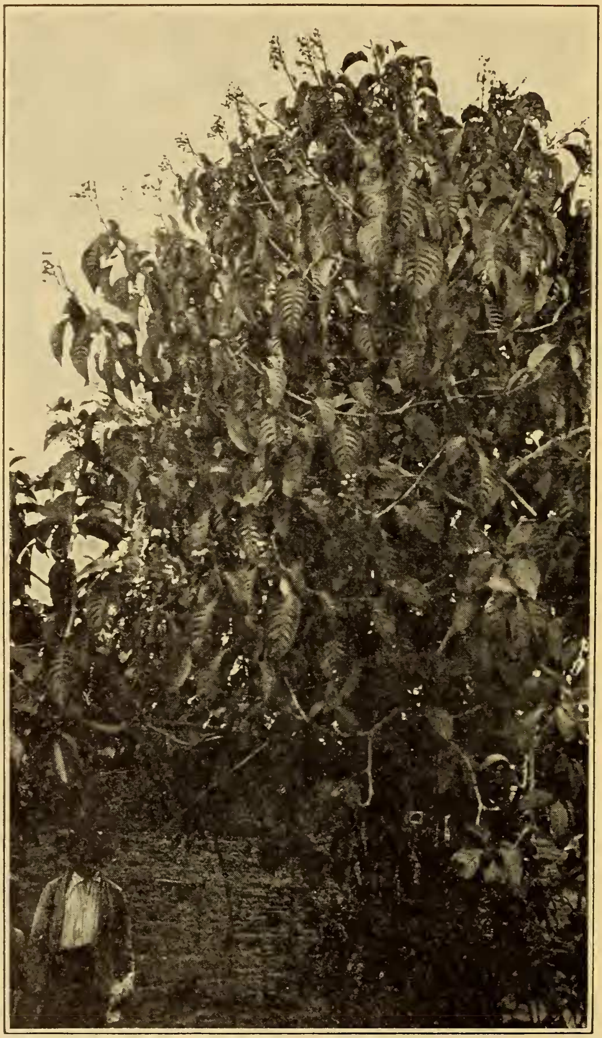

Fig. 1.—*Trichanthera gigantea.* In cultivation at Bucaramanga, Colombia. (Killip and Smith 15452.)

COSTA RICA: Moist forest of Tilarán, Province of Guanacaste, alt. 500 to 650 meters, *Standley and Valerio* 46569 (N).[4] Dry forests of Nicoya, *Tonduz* in 1900 (N).

[4] N = U. S. National Museum; Y = Herbarium of the New York Botanical Garden; G = Gray Herbarium.